

## LOGISTIC FUNCTIONS

Logistic functions can take the following forms

$$f(x) = \frac{N}{1+Ab^{-x}}$$

$$f(x) = \frac{N}{1+Ae^{-kx}} \quad (\text{where } k = \ln b)$$

for some constants  $A$ ,  $N$ , and  $b$  ( $b > 0$  and  $b \neq 1$ ).

In both forms the  $N =$  **the limiting value**

This means when  $x$  is large,  $f(x) \approx N$

In both forms the  **$y$ -intercept**  $= \frac{N}{1+A}$

That means  $f(0) = \frac{N}{1+A}$

### Small values of $x$ and the role of $b$

$$f(x) = \frac{N}{1+Ab^{-x}} \approx f(x) = \left(\frac{N}{1+A}\right) b^x$$

For small  $x$ , the logistic function grows approximately exponentially with base  $b$

### Steps to Calculate a Logistic Function

1. Identify the limiting value
2. Use the  $y$ -intercept formula to solve for constant  $A$ .  $f(0) = \frac{N}{1+A}$
3. Based on the given data pick 2 of the smallest values of  $x$  and their corresponding  $y$  values and plug them into the TI-84/83 calculator
4. Calculate the exponential regression for the two values selected in step 3. In this case the  $b$  value for the exponential regression will be the same for logistic regression. (refer to small values of  $x$  and the role of  $b$ ).

Now you have all the required constants to complete the logistic function for a set of data.

### Steps to Predict $x$ with a Logistic Function

In other words you have the  $f(x)$  but you do not know the  $x$ . To avoid confusion  $f(x) = y$

1. Use the formula  $-x = \log_b\left(\frac{N-y-1}{A}\right)$
2. Based on the law of logarithms this means  $-x = \frac{\log(N-y-1)}{\log(b)} - \frac{\log(A)}{\log(b)}$
3. Make sure to take the absolute value of the answer from step 3, to get the correct predicted value for  $x$ .

**Note:** To predict  $f(x)$  use the logistic function that you calculated in "Steps to Calculate a Logistic Function."