# Derivatives

1. Using limit definition:
   $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
2. Constant:
   $\frac{d}{dx}(c) = 0$
3. Constant Times f(x):
   $\frac{d}{dx}[cf(x)] = cf'(x)$
4. Sum and Difference Rules:
   $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
5. Power Rule:
   $\frac{d}{dx}(x^n) = nx^{n-1}$
6. Product Rule:
   $\frac{d}{dx}[f(x)g(x)] = [f'(x)g(x)] + [f(x)g'(x)]$
7. Quotient Rule (provided $g(x) \neq 0$):
   $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{[f'(x)g(x)]-[f(x)g'(x)]}{[g(x)]^2}$
8. Chain Rule:
   $\frac{d}{dx}f(u) = f'(u)\frac{du}{dx}$
9. Natural Logarithm:
   $\frac{d}{dx}[\ln(u)] = \frac{1}{u}\frac{du}{dx}$
10. Logarithm:
    $\frac{d}{dx}[\log_b(u)] = \frac{1}{u*\ln(b)}\frac{du}{dx}$
11. Exponential:
    $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$
12. $\frac{d}{dx}(b^u) = b^u * \ln(b)\frac{du}{dx}$

# Integrals

1. $\int 1dx = x + C$
2. Power Rule (where $n \neq -1$):
   $\int x^n dx = \frac{x^{n+1}}{n+1} + C,$
3. Special Case of Power Rule (when $n = -1$):
   $\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + C$
4. Exponential:
   $\int e^x dx = e^x + C$
5. $\int b^x dx = \frac{b^x}{\ln(b)} + C$
6. Fundamental Theorem of Calculus:
   $\int_a^b f(x)dx = F(b) - F(a)$
7. U Substitution:
   $\int f(g(x))g'(x)dx = \int f(u)du$, where $u = g(x)$
8. Sum and Difference Rules:
   $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
9. Constant Multiple Rule:
   $\int k * f(x)dx = k\int f(x)dx$, where k constant
10. Integration By Parts:
    $\int u\,dv = uv - \int v\,du$

**Business Formulas**

$Profit = Revenue - Cost$

$Cost = Fixed\ Cost + Variable\ Cost$

$P'(x) = Marginal\ Profit$

$R'(x) = Marginal\ Revenue$

$C'(x) = Marginal\ Cost$

**Derivative by Limit Definition Example:**

$f(x) = 2 + x^2$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{[2+(x+h)^2]-[2+x^2]}{h}$

$= \lim_{h\to 0}\frac{[2+(x+h)(x+h)]-2-x^2}{h}$

$= \lim_{h\to 0}\frac{2+x^2+2xh+h^2-2-x^2}{h}$

$= \lim_{h\to 0}\frac{h(2x+h)}{h}$

$= \lim_{h\to 0} 2x + h$

$= 2x$

**Integral by U Substitution Example:**

$\int xe^{2+x^2}dx$

$let\ u = 2 + x^2$

$then\ du = 2xdx$

$therefore\ \frac{1}{2}du = xdx$

$= \int \frac{1}{2}e^u du$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{(2+x^2)} + C$

**Derivative by Chain Rule Example #1:**

$\frac{d}{dx}(2+x^2)^2$

$= 2(2+x^2) * 2x$

$= 4x(2+x^2)$

$= 8x + 4x^3$

**Chain Rule Example #2:**

$\frac{d}{dx}(e^{(2+x^2)})$

$= e^{(2+x^2)} * 2x$